Instructions:
   a) Closed book, closed notes test, one 8½ inch by 11 inch sheet of handwritten formulas allowed.
   b) 105 minute time limit – strictly enforced.
   c) Show all of your work on problems #6, #7, and #8 (30 points each).

1. A disk and a hoop, each of identical mass $m$ and radius $r$, are released from rest side-by-side on a small hill. After 2 full rotations, which body has a larger angular acceleration? Assume roll without slip.
   a) hoop
   b) disk
   c) the two accelerations are equal
   d) cannot be determined

2. A rigid body is subjected to a horizontal force $P$ as shown. There is friction between the body and the lower surface. The rigid body can
   a) tip with $\alpha > 0$, but not slide.
   b) tip with $\alpha < 0$, but not slide.
   c) slide and/or tip with $\alpha > 0$
   d) slide and/or tip with $\alpha < 0$
   e) slide but not tip

3. The method of instant centers can be used to find
   a) angular accelerations and linear velocities
   b) angular and linear accelerations
   c) linear accelerations and angular velocities
   d) all of the above.

4. The relative velocity ($\vec{v}_{B/A}$) between two points on a rigid body that is rotating is
   a) perpendicular to the position vector $\vec{r}_{B/A}$
   b) parallel to the position vector $\vec{r}_{B/A}$
   c) has components both parallel to and perpendicular to the position vector $\vec{r}_{B/A}$
   d) none of the above.

5. The relative acceleration ($\vec{a}_{B/A}$) between two points on a rigid body that is rotating is
   a) perpendicular to the position vector $\vec{r}_{B/A}$
   b) parallel to the position vector $\vec{r}_{B/A}$
   c) has components both parallel to and perpendicular to the position vector $\vec{r}_{B/A}$
   d) none of the above.
6. The weight of block A is 75 lb. The static coefficient of friction between block A and the horizontal surface is \( \mu_s = 0.60 \), while the dynamic coefficient of friction between block A and the horizontal surface is \( \mu_k = 0.40 \). The mass-moment of inertia about the center of mass for the pulley is \( I_G = 0.42 \text{ slug-ft}^2 = 0.42 \text{ lb-ft-s}^2 \).

a) Find minimum force \( T_{ext} \) required to move block A

b) If \( T_{ext} = 49 \text{ lb} \), find the angular acceleration of the pulley, the linear acceleration of block A and the tension in the horizontal cable.

Bonus: How many revolutions will the pulley make in 2 seconds if it starts from rest with \( T_{ext} = 49 \text{ lb} \)?
7. Bar $AB$ is rotating clockwise with the constant angular velocity of 72 rad/s. The roller at point C is constrained to move only in the horizontal direction. When the mechanism is in the position shown (i.e., side CD is vertical), determine

   a) the angular velocity, $\omega_{BCD}$, of the plate $BCD$ and

   b) the velocity $v_D$ of corner $D$.

\[
\begin{align*}
\vec{r}_{B/A} &= 500 \cos 25^\circ \hat{i} + 500 \sin 25^\circ \hat{j} \\
\vec{r}_{C/B} &= 600 \cos 48.6^\circ \hat{i} + 600 \sin 48.6^\circ \hat{j} \\
\vec{r}_{D/B} &= -900 \hat{j}
\end{align*}
\]

\[
\begin{align*}
AB &= 500 \text{ mm} \\
BC &= 600 \text{ mm} \\
BD &= 600 \text{ mm} \\
CD &= 900 \text{ mm}
\end{align*}
\]

\[
\vec{v}_B = -\omega_{AB} \vec{r}_{B/A} \\
\vec{v}_B = (-72 \text{ rad/s}) \vec{k} \times (500 \cos 25^\circ \hat{i} + 500 \sin 25^\circ \hat{j}) \\
= 15214 \hat{i} - 32627 \hat{j} \text{ m/s}
\]

\[
\vec{v}_D = \vec{v}_B + \omega_{BCD} \times \vec{r}_{C/B} = (15214 \hat{i} - 32627 \hat{j}) + \omega_{BCD} \vec{k} \times (600 \cos 48.6^\circ \hat{i} + 600 \sin 48.6^\circ \hat{j}) \\
= (15214 \text{ m/s} - \omega_{BCD} 450.1 \hat{i}) + (-32627 + 396.8 \omega_{BCD}) \hat{j}
\]

\[
\begin{align*}
0 &= -32627 + 396.8 \omega_{BCD} \\
\omega_{BCD} &= 82.2 \text{ rad/s CCW} \\
v_D &= 21800 \text{ m/s} \hat{i} \leftarrow
\end{align*}
\]

\[
\vec{v}_P = \vec{v}_D + \omega_{BCD} \times \vec{r}_{D/B} = (21800 \text{ m/s}) \hat{i} + (82.2 \text{ rad/s}) \vec{k} \times (-900 \hat{j}) \\
= -21800 \hat{i} + 73980 \hat{j} = 52200 \text{ m/s} \hat{i}
\]

30\degree
8. The wheel rolls without slipping on the horizontal surface at point O. In the position shown, the angular velocity of the wheel, \( \omega_w \), is 3.6 rad/s counterclockwise (CCW), and its angular acceleration, \( \alpha_w \), is 5.1 rad/s\(^2\) clockwise (CW). At this instant, the angular velocity of bar AB, \( \omega_{AB} \), is 2.7 rad/s counterclockwise (CCW).

YOU DO NOT NEED TO SOLVE FOR ANY OTHER VELOCITIES!

For this position, determine:

a) the angular acceleration, \( \alpha_{AB} \) of rod AB and
b) the linear acceleration of slider B, \( a_B \).

\[
\vec{\alpha}_A = \alpha_w \vec{r}_A \hat{\imath} - \omega_w^2 \vec{r}_A \hat{j}
\]
\[
\vec{a}_A = (3.1 \text{ rad/s}^2) (1.5 \text{ ft}) \hat{\imath} - (3.6 \text{ rad/s}^2) (1.5 \text{ ft}) \hat{j}
\]
\[
= 7.65 \text{ ft/s}^2 \hat{\imath} - 19.44 \text{ ft/s}^2 \hat{j}
\]

\[
\vec{\alpha}_B = \vec{\alpha}_A + \alpha_{AB} \times \vec{r}_A - \omega_{AB}^2 \vec{r}_A
\]
\[
\vec{a}_B = \left( \begin{array}{c} 15.3 \text{ ft/s}^2 \hat{\imath} - 19.44 \text{ ft/s}^2 \hat{j} \\ 41/4 \end{array} \right) + \left( \begin{array}{c} \frac{3}{5} \alpha_{AB} \hat{\imath} + \left( \frac{2}{5} \alpha_{AB} \right) \end{array} \right) - (2.7 \text{ rad/s}^2) \left( \frac{3}{5} \alpha_{AB} \hat{\imath} + \left( \frac{2}{5} \alpha_{AB} \right) \right)
\]

\[
\vec{a}_B = (7.65 \text{ ft/s}^2 \hat{\imath} + \frac{4}{5} \alpha_{AB} - 4.32 \text{ ft/s}^2) \hat{\imath} + (-19.44 \text{ ft/s}^2 + \frac{3}{5} \alpha_{AB} + 5.83 \text{ ft/s}^2) \hat{j}
\]

\[
0 = 3.270 \text{ ft/s}^2 + \frac{4}{5} \alpha_{AB} \quad \alpha_{AB} = -4.09 \text{ rad/s}^2 \quad \frac{d}{s^2} = \boxed{4.10 \text{ rad/s}^2} \text{ CW}
\]

\[
a_{BC} = -13.408 \text{ ft/s}^2 + \frac{3}{5} (4.10 \text{ rad/s}^2)
\]
\[
= -16.068 \text{ ft/s}^2 = \boxed{16.07 \text{ ft/s}^2} \text{ down}
\]
\[
22/30
\]