Short answer

2. (4 pts) For real flow over a cylinder,

- Which has a streamline pattern more similar to inviscid flow: Re \( \sim 1 \) or Re \( \sim 1,000,000 \)?

\[ \text{Re} \sim 1 \]

- For Re \( \sim 1 \) flow over a cylinder, is the drag dominated by pressure drag or friction drag?

\[ \text{Pressure drag} \]

A

(5 pts) What are the three primary assumptions for ideal (potential) flow? List any kinematic relationships involving the velocity field that can be associated with these assumptions.

\[ \sqrt{\text{inviscid}} - \frac{dv}{dx} = 0 \]
\[ \sqrt{\text{irrotational}} - \nabla \times \vec{V} = 0 \checkmark \]
\[ \sqrt{\text{incompressible}} - \frac{\partial \rho}{\partial t} = 0 \]

A

(4 pts) Write the equation that represents the vortex sheet distribution for a thin, symmetric airfoil. Define each term.

\[ \gamma(\theta) = 2 \times V_\infty \frac{1 + \cos \theta}{\sin \theta} \]

\( \alpha = \text{angle of attack} \checkmark \)
\( V_\infty = \text{free-stream velocity} \checkmark \)
\( \theta = \text{angular distance from the leading edge} \checkmark \)
\( r = ? \)

(2 pts) List two types of boundary conditions commonly applied for ideal flow.

\[ \begin{align*}
\zeta &= 0 \checkmark \\
\frac{\partial V}{\partial t} &= 0 \times
\end{align*} \]

(5 pts) True or False

1. \( \checkmark \) / F: A source/sink panel method predicts zero lift.

2. \( \checkmark \) / F: The Kutta condition states that the flow velocity at the TE is zero.

T / \( \checkmark \) Ideal flow predicts an adverse pressure gradient on the aft side of the cylinder.

4. \( \checkmark \) / F: Laminar flow airfoils generally have the maximum thickness moved toward the leading-edge.

5. \( \checkmark \) / F: Thin airfoil theory predicts that the quarter-chord is the center of pressure for cambered airfoils.
(5 pts) What elementary flows would you use to model a spinning cylinder in a flow and how could you estimate the magnitude of force, in any, the spinning cylinder would create based on the strength of the elements?

For a spinning cylinder, to create the shape of the cylinder itself, we use a source and sink with a fixed stream velocity flowing towards it. Then with an added vortex, the "spin" can be modeled. To estimate the magnitude of lift, you can use the Bernoulli's equation given the velocity on both the upper and lower surface. Does Bernoulli yield force?

Bernoulli yields $p$ via $V$. $V$ from $\psi$ or $\phi$ field.

Then must integrate.

Or

\[ L' = \rho U_0 \Gamma \]

(5 pts) For linear Couette flow with a plate velocity of $U$ and a gap height of $h$, define a stream function and velocity potential if possible.

\[
\begin{align*}
\psi_x &= \frac{U}{h} y, \quad \psi_y = 0
\end{align*}
\]

\[
\begin{align*}
\phi_x &= \frac{\phi}{\partial y} & \phi_x &= \frac{\phi}{\partial z}
\end{align*}
\]

\[
\begin{align*}
\chi \phi &= \int \frac{\psi}{h} y dy = \frac{U}{2h} y^2, \quad \text{Should } \phi \text{ exist?}
\end{align*}
\]

\[
\begin{align*}
\psi &= \int \frac{\psi}{h} y d\theta = \frac{U}{h} y \theta, \quad \text{Does } \psi \text{ exist?}
\end{align*}
\]

\[
\begin{align*}
\phi &\rightarrow \psi
\end{align*}
\]

\[
\chi = \frac{U}{2} y^2
\]
2. (20 pts)

The MCL of a negative-cambered airfoil is modeled with two straight segments as shown below. For the given geometry use the results from thin-airfoil theory to determine the lift and drag coefficients at 6°.

\[ \frac{dz}{dx} = \frac{c/15 - 0}{c/4 - 0} = -0.0533 \]

\[ \frac{d^2z}{dx^2} = \frac{0 - \frac{c}{5}}{c - 3c/4} \]

\[ \alpha = 6° \]

\[ \chi = \frac{c}{3} (1 - \cos \phi) \]

\[ \Theta^* = \cos \phi (1 - \frac{3c}{4}) = 2.094 \text{ rad} \]

\[ \delta = \frac{1}{\pi} \int_{0}^{\phi} \left( -0.0533 \right) (\cos \theta - 1) \, d\theta + \int_{\phi}^{\pi} \left( -0.16 \right) (\cos \theta - 1) \, d\theta \]

\[ \delta = 0.11828 \text{ rad} \]

\[ C = 2\pi \left( 0.11828 + 0.11828 \right) = 1.4 \]

\[ C_d = \frac{C}{4} \]

\[ C_d = 0.35 \]
Two sources of equal strength (\( \Lambda = 10 \text{ m}^2/\text{s} \)) are placed 2 m apart from each other. Determine an equation for the velocity of the induced flow along a line that bisects and is perpendicular to the axis that runs through the origins of both sources (see figure). Sketch the velocity magnitude along the bisecting line, labeling important points. Hint: exploit symmetry when advantageous.