AEM 368

SPRING 2011

QUIZ-1

12:30 PM-1:45 PM

Instructions:

1. This QUIZ is open-book and open-notes, the calculator is allowed to use.
2. No cooperation is allowed, please see the instructor if you have any questions.
3. Please return the QUIZ-1 sheets along with your solutions.

Signature: ___________________________  Date: 2/8/11

1. 30 + 0
2. 18 x 0
3. My calculator died halfway through the test, that's why the computations aren't complete.
Problem 1 (30+10 Points)

A) Assuming the linear temperature variation, "CALCULATE" the standard atmosphere values of $P$ and $\rho$ at a geopotential altitude of 11Kms, from sea level.

$T_h=11Kms = 216.66K$

If an instrument mounted on a wing tip measures a total pressure of $2.382\times10^4 \text{ N/m}^2$, find the true airspeed and equivalent airspeed.

$$ p = \rho \left( \frac{T}{T_1} \right)^{\frac{y}{R}} = (101,325) \left( \frac{288,15}{216.66} \right)^{\frac{4.81}{1.667}} = 22619.8\text{ Pa} $$

$$ \rho = \frac{p}{RT} = \frac{22619.8}{(287)(216.66)} = 0.3638 \text{ kg/m}^3 $$

$$ p_1 + \frac{1}{2} \rho V_1^2 = p_2 + \frac{1}{2} \rho V_2^2 $$

$$ 22619.8 + 0 = 28820 \rho_2 + \frac{1}{2} (0.3638 \text{ kg/m}^3) V_2^2 $$

$$ V_{tAS} = V_2 = 80.758 \text{ m/s} $$

$$ V_{TAS} = \frac{V_{tAS}}{\sqrt{\frac{\rho_1}{\rho_2}}} = \frac{80.758}{\sqrt{0.3638/1.225}} = 148.191 \text{ m/s} $$

B) If airspeed vector $\vec{V}_a = [u, v, w]^T$ has magnitude $V_a$, find the velocity components $[u, v, w]$ expressed in terms of new variables $V_a, \alpha$ and $\beta$ (magnitude, angle of attack and side slip angle) only.

$$ V_a = \sqrt{u^2 + v^2 + w^2} $$
Problem 2 (30+10 Points)

A) The orientation of airplane is given using 3-2-1 Euler angles with \(-30^\circ, 15^\circ\) and \(45^\circ\) about the respective axes. The airplane's angular velocity and velocity is expressed in the body fixed frame as \([0.2, 0.1, 0]^T\) rad/sec and \([20, 85, -20]^T\) ft/sec.

i) Find velocity of airplane observed in the Earth-fixed frame.

ii) Find the time rate of change of Euler angles.

\[
\begin{align*}
\alpha &= -30^\circ, \\
&= 0.5236
\end{align*}
\]

\[
\beta = 15^\circ, \\
&= 0.262
\]

\[
\gamma = 45^\circ, \\
&= 0.785
\]

\[
C_3 = \begin{bmatrix}
\cos \alpha & \sin \alpha & 0 \\
-sin \alpha & \cos \alpha & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

\[
C_2 = \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \beta & \sin \beta \\
0 & -\sin \beta & \cos \beta
\end{bmatrix}
\]

\[
C_1 = \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \gamma & -\sin \gamma \\
0 & \sin \gamma & \cos \gamma
\end{bmatrix}
\]

\[
V = C_1 C_2 C_3 V_b = \begin{bmatrix}
\cos (\alpha) (20 \cos \gamma + 85 \sin \gamma) + 20 \sin \gamma \\
\cos (\alpha) 85 \sin \gamma - 20 \sin \beta + \sin \alpha \sin \beta (20 \cos \gamma + 85 \sin \gamma) - 20 \cos \beta \sin \gamma \\
-20 \cos (\alpha) \cos (\beta) + \sin (\alpha) \cos (\beta) (20 \cos \gamma + 85 \sin \gamma) + \sin (\alpha) \sin (\beta) \cos (\beta) (20 \cos \gamma + 85 \sin \gamma)
\end{bmatrix}
\]

\[
= \begin{bmatrix}
(0.166)(20(0.707)) + (85(0.707)) + 20(-0.5) \\
(0.166)(85(0.707)) - 20(0.259) + \sin
\end{bmatrix}
\]

\[90\°C\text{C}
\]

B) Prove that the angular momentum vector \(\mathbf{H}\) of an airplane is the same whether or not the wind vector \(\mathbf{W}\) is zero.
**Problem 3 (20 Points)**

The general nonlinear pitching moment equation of motion, of an airplane (assumed as a rigid body) is

\[ M = I_x \dot{q} + rp(I_z - I_x) + I_{xz}(p^2 - r^2) - I_{xy}(\dot{\psi} + qr) + I_{yz}(pq - \dot{\phi}); \]

\[ p, q, r = f(\psi, \theta, \phi, \dot{\psi}, \dot{\theta}, \dot{\phi}) \]

Determine the linearized equation of pitching moment equation for a given reference flight condition.

Reference: Steady, wings level, unaccelerated flight.

\[ \phi, \dot{\phi}, \theta, \dot{\theta}, \psi, \dot{\psi} = 0; \quad p, q, r = 0 \]

\[ M = \frac{\partial M}{\partial \psi} \Delta \psi + \frac{\partial M}{\partial \theta} \Delta \theta + \frac{\partial M}{\partial \phi} \Delta \phi + \frac{\partial M}{\partial \dot{\psi}} \Delta \dot{\psi} + \frac{\partial M}{\partial \dot{\theta}} \Delta \dot{\theta} + \frac{\partial M}{\partial \dot{\phi}} \Delta \dot{\phi} \]

\[ M = \frac{\partial M}{\partial \psi} \Delta \psi + \frac{\partial M}{\partial \theta} \Delta \theta \]

\[ M = \frac{\partial M}{\partial \phi} \Delta \phi + \frac{\partial M}{\partial \dot{\phi}} \Delta \dot{\phi} \]

\[ M = \psi (r \dot{\phi} - I_x) + I_{xy} (\dot{\psi} - r) - I_{xz} (\dot{\theta} - p) \]

\[ \rho = \dot{\phi} - \psi \dot{\theta} = 0 \quad q = \dot{\theta} C_t + \dot{\phi} C_t C_\phi \quad r = \dot{\phi} C_t C_\phi - \dot{\theta} S_\phi \]