Problem 1 (30 points):

Air flows through a converging-diverging duct designed to achieve $M=1.5$ at the exit under fully expanded conditions. The air discharges to the atmospheric conditions with $P_{atm} = 1$ atm. Calculate the total pressure, $P_0$, required in the plenum chamber under the following conditions:

a) (10 pts) The flow is fully expanded.
b) (10 pts) There is a normal shock right at the exit.
c) (10 pts) The flow is overexpanded and the flow turns by 5 degrees passing over an oblique shock.

Problem 2 (30 points):

Air flows through a duct as shown below.

a) (20 pts) Calculate the Mach numbers in sections 2, 3 and 4.
b) (10 pts) If static pressure in section 1 is, $P_1=1$atm, calculate the pressure, $P_4$. 

---

Open book. No additional notes allowed. Be specific indicating the equations and tables used. Write down the equation number and/or the tables used.
Problem 3 (40 points): The flow is turned using a compression ramp and an expansion corner. If the approach flow Mach number is, $M=3$, approach flow static pressure is, $P_1 = 1$ atm, and the ramp angle is 10 degrees,

a) (10 pts) Calculate the Mach number after the compression ramp.
b) (10 pts) Calculate the flow turning angle, $\theta$, required to achieve $M=3$ after the expansion corner.
c) (10 pts) Calculate the total pressure after the compression corner.
d) (10 pts) Calculate the static pressure after the expansion corner.


\[ P_0 \]

Given: \( M = 1.5 \), \( P_{\infty} = 1 \text{ atm} \)

a. \( \frac{A}{A_\infty} = \left( \frac{1}{M^2} \left[ \frac{2}{\delta+1} \left( 1 + \frac{\delta-1}{2} M^2 \right) \right]^{\frac{\delta+1}{\delta-1}} \right)^{\frac{1}{2}} \)

\[ = \left( \frac{1}{1.5^2} \left[ \frac{2}{2.4} \left( 1 + \frac{0.4}{2} \cdot 1.5^2 \right) \right]^{\frac{2.4}{0.4}} \right)^{\frac{1}{2}} = 1.126 \]

From A.1,

\[ \frac{P_0}{\rho} = 3.671 \]

\[ P_0 = \frac{P_0}{\rho} \cdot \rho = (3.671)(1) = 3.671 \text{ atm} \]

b. \( \frac{P_0}{P_\infty} = 3.671 \quad \frac{P_\infty}{P_\infty} = 1 \Rightarrow P_\infty = 1 \text{ atm} \)

\[ P_0 = (3.671) \left( \frac{1}{2.458} \right)(1) = 1.49 \text{ atm} \]

c. \( P_0 = \frac{P_0}{P_\infty} \frac{P_\infty}{P_\infty} \)

\[ = (3.671)(1) \]

\[ M_\infty = M, \sin \theta = 1.5, \sin 50° = 0.769 \]

\[ \frac{P_\infty}{P_{\infty}} = 0.305 \]

\[ -8 \]
\( \text{Problem: } M_2, M_3, M_4, P_4 \)

\( \text{Given: } M_1 = 2, \theta = 0^\circ, P_1 = 1 \)

\( \frac{P_0}{P_1} = 7.824 \)

For \( M_1 \):

\( V_1 = 26.38^\circ \)

\( V_2 = 26.38 + 8 = 34.38^\circ \)

\( M_2 = 2.3 \)

\( V_3 = 34.38 + 8 = 42.38^\circ \)

\( M_3 = 2.65 \)

\( V_4 = 42.38 + 8^\circ = 50.38^\circ \)

\( M_4 = 3.05 \)

\[ P_4 = \frac{P_1}{P_{41}} \frac{P_{42}}{P_{41}} \frac{P_{42}}{P_{41}} \frac{P_{41}}{P_1} = \left( \frac{1}{39.59} \right) \left( 1 \right) \left( 1 \right) \left( 7.824 \right) \left( 1 \right) \]

\[ = 0.198 \text{ atm} \]
3. Find: $M_e$, $\theta_e$, $P_{02}/P_1$

Given: $M_i = 3$, $P_1 = 1$, $\theta_i = 10^\circ$

$\beta = 27.5^\circ$

$M_{ni} = M_i \sin \beta = 3 \sin 27.5^\circ = 1.388$

$\frac{P_{02}}{P_1} = 2.055 \quad P_2 = 2.055 \times 1 \quad \frac{P_{02}}{P_1} = 2.980$

$M_e = 0.7483$

$M_e = \frac{M_{ni}}{\sin(\beta + \theta_i)} = \frac{0.7483}{\sin(22.5 - 10)} = \frac{0.7483}{\sin 12.5^\circ} = 2.488$

$M_j = 3 \quad V_3 = 49.76^\circ$

$M_e = 2.488 \quad V_2 = 38^\circ$

$\theta_e = V_3 - V_2 = 49.76^\circ - 38^\circ = 11.76^\circ$

$c. \quad P_{02} = \frac{P_{02}}{P_1} \quad P_1 = (2.980)(1.001) = 2.980 mN$

$d. \quad P_3 = \frac{P_3}{P_{02}} \quad P_{02} = \frac{P_{02}}{P_1} \quad P_2 = \frac{P_2}{P_1}$

$P_3 = \left(\frac{1}{15.81}\right) \left(\frac{2.980}{2.055}\right) (2.055)$

Error due to $P_{02}$

$P_3 = 0.188 \text{ atm}$

$\frac{P_{02}}{P_1} = \frac{P_{02}}{P_01}$

From $M_i = 3$, $\theta_i = 10^\circ$

$P_1 = 0.963$

we would expect a much higher number than $2.980$

Can not use it because we do not have $P_0$.

Because we do not have $P_0$, we cannot use the formula.

Because we do not have $P_0$, we cannot use $\frac{P_{02}}{P_1}$.