1. Sketch the graph of $z = f(x, y) = 4 - x^2$

2. Find the equation of the trace of the surface given by $4y^2 = x^2 + z^2$ in the plane $z = k$. Write your answer in standard form.

   \[
   \frac{4y^2}{k^2} - \frac{z^2}{k^2} = 1
   \]

3. Find the vector function that represents the curve of intersection of the surfaces $z = x^2$ and $x^2 + y^2 = 1$

   \[
   \vec{F}(t) = \langle \cos t, \sin t, \cos^2 t \rangle
   \]

   (OR)

   \[
   \vec{F}(t) = \langle \sin t, \cos t, \sin^2 t \rangle
   \]

   From $x^2 + y^2 = 1$ we get $x = \cos t$ and $y = \sin t$

   Then $z = x^2 = \cos^2 t$
7. Find parametric equations for the tangent line to the curve $r(t) = \langle \ln t, 3t, t^2 \rangle$ at the point $(0, 3, 1)$, so $t = 1$ gives the point $(0, 3, 1)$.

\[
\mathbf{r}'(t) = \langle \frac{1}{t}, 3, 2t \rangle
\]

and

\[
\mathbf{r}(1) = \langle 1, 3, 2 \rangle
\]

\[
\mathbf{r}'(1) = \langle 0, 3, 2 \rangle
\]

\[
\mathbf{x} = 0 + 1t \quad , \quad \mathbf{y} = 3 + 3t \quad , \quad z = 1 + 2t
\]

8. Find the length of the curve $r(t) = \langle 2t, \frac{4}{3}t^{3/2}, \frac{1}{2}t^2 \rangle$ with $0 \leq t \leq 1$.

\[
\mathbf{r}'(t) = \langle 2, \frac{4}{3}t^{1/2}, t \rangle
\]

\[
|\mathbf{r}'(t)| = \sqrt{4 + \frac{16t}{3} + t^2} = \sqrt{(t+2)^2} = t + 2
\]

\[
L = \int_{0}^{1} |\mathbf{r}'(t)| \, dt = \int_{0}^{1} (t+2) \, dt = \left. \frac{1}{2} t^2 + 2t \right|_{0}^{1} = \frac{5}{2}
\]

9. Reparametrize the curve $r(t) = \langle 4t, 1-4t, 2t \rangle$ with respect to arc length measured from the point where $t = 0$ in the direction of increasing $t$.

\[
\mathbf{r}'(t) = \langle 4, -4, 2 \rangle
\]

\[
|\mathbf{r}'(t)| = \sqrt{16 + 16 + 4} = \sqrt{36} = 6
\]

\[
S = \int_{0}^{t} |\mathbf{r}'(u)| \, du = \int_{0}^{t} 6 \, du = 6t\big|_{0}^{t} = 6t
\]

\[
\mathbf{r}(S) = \langle 4\left(\frac{S}{6}\right), 1-4\left(\frac{S}{6}\right), 2\left(\frac{S}{6}\right) \rangle = \langle \frac{2S}{3}, 1-\frac{2S}{3}, \frac{5S}{3} \rangle
\]

10. Find the curvature when $r(t) = \langle t, t, 1+t^2 \rangle$.

\[
\mathbf{r}'(t) = \langle 1, 1, 2t \rangle \quad |\mathbf{r}'(t)| = \sqrt{1 + 1 + 4t^2}
\]

\[
\mathbf{r}''(t) = \langle 0, 0, 2 \rangle
\]

\[
\mathbf{r}'(t) \times \mathbf{r}''(t) = \begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
1 & 1 & 2t \\
0 & 0 & 2 \\
\end{vmatrix} = \langle 2, -2, 0 \rangle
\]

\[
\kappa(t) = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3} = \frac{2\sqrt{2}}{(2 + 4t^2)^{3/2}}
\]
PART II - INSTRUCTIONS: Give a complete solution and show all work in the space provided below each problem. Each problem on Part II counts 10 points and partial credit is possible.

11. (10 points) Draw both a contour map and a graph of the function

\[ z = f(x, y) = \sqrt{x^2 + y^2} \]

\[ z = k \Rightarrow x^2 + y^2 = k \]

12. (10 points) Show that \( \lim_{(x,y) \to (0,0)} \frac{xy^2}{x^2 + y^4} \) does not exist.

Let \( x = 0 \). Then \( \lim_{(x,y) \to (0,0)} \frac{xy^2}{x^2 + y^4} = \lim_{(x,y) \to (0,0)} \frac{0 \cdot y^2}{0^2 + y^4} = \lim_{(x,y) \to (0,0)} 0 = 0 \)

Let \( x = y^2 \). Then \( \lim_{(x,y) \to (0,0)} \frac{xy^2}{x^2 + y^4} = \lim_{(x,y) \to (0,0)} \frac{y^2 \cdot y^2}{y^4 + y^4} = \lim_{(x,y) \to (0,0)} \frac{y^4}{2y^4} = \lim_{(x,y) \to (0,0)} \frac{1}{2} = \frac{1}{2} \)

Since these limits are not equal, the given limit does not exist.